Cambridge International Examinations
Cambridge Pre-U Certificate

MATHEMATICS (PRINCIPAL) **9794/01**

Paper 1 Pure Mathematics **May/June 2018**

2 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document consists of **3** printed pages and **1** blank page.

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1 Solve $5x + 3 < |3x - 1|$. [4]

2 It is given that $\mathrm{f}(x) = 4 + 3\sqrt{x}$, where $x \geqslant 0$.

(i) State the range of f. [1]

(ii) State the value of $\mathrm{ff}(16)$. [1]

(iii) Find $\mathrm{f}^{-1}(x)$. [2]

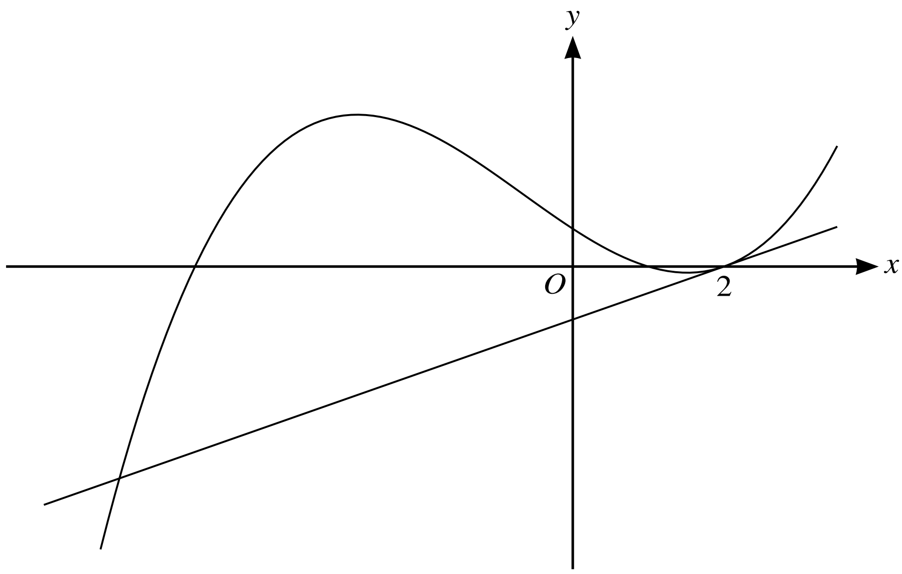
(iv) On the same axes, sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ and state how the graphs are related. [3]

3 Given that $z = 1$ is the real root of the equation $z^3 - 1 = 0$, find the two complex roots. [4]

4 **(a)** Sketch the graph of $y = \sec\theta$ for $0 \leqslant \theta \leqslant 2\pi$. [1]

(b) Solve $\sec\theta = \operatorname{cosec}\theta$ for $0 \leqslant \theta \leqslant 2\pi$. [4]

5



The diagram shows the curve with equation $y = x^3 + 2x^2 - 13x + 10$ and the tangent to the curve at the point $(2, 0)$.

(i) Find the equation of this tangent and verify that the tangent intersects the curve when $x = -6$. [4]

(ii) Calculate the exact area of the region bounded by the curve and the tangent. [6]

6 Two straight lines have equations

$$\mathbf{r} = -3\mathbf{i} + 11\mathbf{j} - 9\mathbf{k} + \lambda(4\mathbf{i} + 7\mathbf{j} + 8\mathbf{k})$$

and

$$\mathbf{r} = 21\mathbf{i} + 2\mathbf{j} + 15\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} + 4\mathbf{k}).$$

(i) Show that the lines intersect and find the coordinates of their point of intersection. [5]

(ii) Find the acute angle between the two lines. [3]

7 Find the coordinates of the two stationary points of the curve

$$9x^2 + 4y^2 - 6x - 4y = 34,$$

showing that one is a maximum and one is a minimum. [8]

8 **(i)** Using the quotient rule, show that $\dfrac{\mathrm{d}}{\mathrm{d}\theta}(\tan 3\theta) = 3 + 3\tan^2 3\theta$ for $-\frac{1}{6}\pi < \theta < \frac{1}{6}\pi$. [3]

(ii) Hence find the value of $\displaystyle\int_{\frac{1}{12}\pi}^{\frac{1}{9}\pi} \tan^2 3\theta \,\mathrm{d}\theta$, giving your answer in the simplest exact form. [5]

9 **(i)** Find the coordinates of the stationary point of the curve with equation

$$y = \ln x - kx, \text{ where } k > 0 \text{ and } x > 0,$$

and determine its nature. [4]

(ii) Hence show that the equation $\ln x - kx = 0$ has real roots if $0 < k \leqslant \dfrac{1}{\mathrm{e}}$. [2]

(iii) In the particular case that $k = \frac{1}{3}$, the equation $\ln x - kx = 0$ has two roots, one of which is near $x = 5$.

Use the Newton-Raphson process to find, correct to 3 significant figures, the root of the equation $\ln x - \frac{1}{3}x = 0$ which is near $x = 5$. [3]

(iv) Show that the equation $\ln x - kx = 0$ has one real root if $k \leqslant 0$. [3]

(v) Explain why the equation $\ln x - kx = 0$ has two distinct real roots if $0 < k < \dfrac{1}{\mathrm{e}}$. [2]

10 **(i)** Using partial fractions, find the general solution of the differential equation

$$2\frac{\mathrm{d}y}{\mathrm{d}x} = y - y^3 \text{ for } 0 < y < 1,$$

giving your solution in the form $y = \mathrm{f}(x)$. [9]

(ii) Determine $\lim\limits_{x \to -\infty} \mathrm{f}(x)$ and $\lim\limits_{x \to +\infty} \mathrm{f}(x)$. [3]

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